

ЭНТРОПИЯ ЧЕРНОЙ ДЫРЫ В МОДЕЛИ РЕЙСНЕРА – НОРДСТРЁМА – ДЕ СИТТЕРА / ENTROPY OF REISSNER–NORDSTRÖM–DE SITTER BLACK HOLE

Аннотация. Настоящая работа посвящена развитию макроскопических методов анализа физики высоких энергий. В практически важном случае черной дыры в релятивистской космологической модели (вселенной де Ситтера) рассмотрено описание ее эволюции в рамках феноменологического подхода, аналогичного классической термодинамике, когда площадь черной дыры играет роль энтропии, а поверхностная гравитация, соответственно, роль температуры. Предметом исследования стали способы расчета эффективных термодинамических величин черной дыры. При подсчете энтропии черной дыры использована гипотеза взаимозависимости горизонта событий и космологического горизонта.

Для решения поставленной задачи использованы системный и структурно-функциональный подходы, методы космологии, релятивистской механики и геометрической теории тяготения Эйнштейна, в частности, точные решения уравнений общей теории относительности с космологической постоянной для метрики Рейснера – Нордстрёма при описании пространства-времени.

Найдено аналитическое решение для подсчета полной энтропии сферически симметричной статичной заряженной черной дыры в модели Рейснера – Нордстрёма для пространства де Ситтера. Показано, что выражение для энтропии включает в себя не только сумму энтропий горизонта событий черной дыры и космологического горизонта, но также дополнительный член, учитывающий перепутывание между ними. Полученные результаты термодинамики черных дыр углубляют аналогию с первым началом классической термодинамики, что расширяет применимость подхода для космологических исследований.

Ключевые слова: космология, общая теория относительности, вселенная де Ситтера, метрика Рейснера – Нордстрёма, физика черных дыр, горизонт событий, космологический горизонт, поверхностная гравитация, энтропия.

Abstract. The paper studies the development of the macroscopic methods of high-energy physics analysis. The authors consider the evolution of black holes within the phenomenological approach, analogous to classical thermodynamics, in which the black hole area determines its entropy, and the surface gravitation, correspondingly, - the temperature, in the framework of the relativist cosmological model (de Sitter universe). The research subject is the ways of calculation of effective thermodynamic properties of black holes. To calculate a black hole entropy, the authors apply the event horizon and cosmological horizon interdependence hypothesis. To accomplish the research task, the authors apply the system and structural-functional approaches, the methods of cosmology, relativistic mechanics and Einstein's geometric theory of gravitation, in particular, the exact solutions of the Einstein field equations with the cosmological constant for the Reissner- Nordström metric for the space-time description. The authors find the analytical solution for the calculation of the total entropy of a spherically symmetric charged black hole in the Reissner- Nordström model for de Sitter universe. The paper shows that the expression for entropy includes not only the sum of entropies of the event horizon and cosmological horizon of the black hole, but also the additional term, taking into account their entanglement. The obtained results of black hole thermodynamics extend the analogy with the first law of thermodynamics, thus broadening the applicability of the approach to the cosmological studies.

Keywords: Black holes Physics, Reissner–Nordström metric, de Sitter space, General relativity theory, Cosmology, Theoretical Physics, Event horizon, Cosmological horizon, Entropy, Surface gravity.

The astronomical observations show that our Universe is probably approaching de Sitter one [1-4]. However, as is well known that in de Sitter space there is no spatial infinity and no asymptotic Killing vector which is globally timelike [5]. Moreover, black holes in de Sitter space cannot be in thermodynamic equilibrium in general. Because there are multiple horizons with different temperatures for de Sitter black holes. To overcome this problem, one can analyze one horizon and take another one as the boundary or separate the two horizons by a thermally opaque membrane or box [6-7]. In this way, the two horizons can be analyzed independently. Besides, one can also take a global view to construct the globally effective temperature and other effective thermodynamic quantities [8-9]. No matter which method is used, the total entropy of de Sitter black hole is supposed to be the sum of both horizons, namely $S=S_b+S_c$ [10].

We think that the truth may be not so simple because the event horizon and the cosmological horizon are not independent. There may exist some correlations between them due to the following considerations. We can take the Reissner–Nordström–de Sitter (RNdS) black hole as example. There are first laws of thermodynamics for both horizons. According to [11], the first laws for the black hole horizon and the cosmological horizon are respectively:

$$\begin{aligned} dM &= T_b dS_b + \Phi_b dQ + V_b d\Lambda, \\ dM &= -T_c dS_c + \Phi_c dQ + V_c d\Lambda, \end{aligned} \tag{1}$$

where M is the conserved mass in dS space, Φ and Q stand for the electric potential and electric charge, and Λ is the cosmological constant. Including the variable Λ , the above first laws can have corresponding Smarr formulae, which are also given in [11]. The two laws in Eq. (1) are not truly independent. They depend on the same quantities M, Q, Λ . All the geometric and thermodynamic quantities for the both horizons can be represented by M, Q, Λ . Therefore, the size of black hole horizon is closely related to the size of the cosmological horizon, and the evolution of black hole horizon will lead to the evolution of the cosmological horizon.

Considering the correlation or entanglement between the event horizon and the cosmological horizon, the total entropy of the RNdS black hole is no longer simply $S=S_b+S_c$, but should include an extra term from the contribution of the correlations of the two horizons.

The line element of the RNdS black holes is given by

$$ds^2 = -h(r)dt^2 + h(r)^{-1}dr^2 + r^2d\Omega^2, \tag{2}$$

where

$$h(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2, \tag{3}$$

There are three positive real roots for $h(r)=0$. The smallest one r_- is the inner/Cauchy horizon, the intermediate one r_+ is the event horizon of black hole and the largest one is the cosmological horizon.

Generally, the temperatures at the event horizon and cosmological horizon are not equal. Thus, the whole RNdS system cannot be in equilibrium thermodynamically. However, there are two special cases for the RNdS black hole, in which the temperatures at the both horizons are the same. One case is the so-called Nariai black hole, the other is the lukewarm black hole [12-15]. For the Nariai black hole, the event horizon and cosmological horizon coincide *apparently* and have the same temperature, zero or nonzero [16].

Considering the connection between the black hole horizon and the cosmological horizon, we can derive the effective thermodynamic quantities and corresponding first law of black hole thermodynamics*:

* One can also take the cosmological constant Λ as the pressure and then derive the effective volume. In this case the effective first law should be: $dM=T_{\text{eff}}dS+\phi_{\text{eff}}dQ+V_{\text{eff}}dP$. This has been done in another paper.

$$dM = T_{eff} dS - P_{eff} dV + \Phi_{eff} dQ \quad (4)$$

Here the thermodynamic volume is that between the black hole horizon and the cosmological horizon, namely

$$V = \frac{4\pi}{3} (r_c^3 - r_+^3) = \frac{4\pi}{3} r_c^3 (1 - x^3), \quad (5)$$

where $x = \frac{r_+}{r_c}$.

The total entropy can be written as

$$S = S_+ + S_c + S_{ex} = \pi r_c^2 [1 + x^2 + f(x)]. \quad (6)$$

Here the undefined function $f(x)$ represents the extra contribution from the correlations of the two horizons. Then, how to determine the function $f(x)$?

In general cases, the temperatures of the black hole horizon and the cosmological horizon are not the same, thus the globally effective temperature T_{eff} cannot be compared with them. However, in the special case, such as lukewarm case, the temperatures of the two horizons are the same. We conjecture that in this special case the effective temperature should also take the same value. On the basis of this consideration, we can obtain the information of $f(x)$.

According to Eq. (4), the effective temperature can be derived by

$$T_{eff} = \left(\frac{\partial M}{\partial S} \right)_{Q,V} = \frac{\frac{1}{r_c} \left(\frac{\partial M}{\partial x} \right)_{r_c} (1 - x^3) + \left(\frac{\partial M}{\partial r_c} \right)_x x^2}{2\pi r_c [(2x + f'(x))(1 - x^3) / 2 + (1 + x^2 + f(x))x^2]}. \quad (7)$$

From Eq. (3), one know that

$$M = \frac{(x+1)(x^2 r_c^2 + Q^2 x^2 + Q^2)}{2x(x^2 + x + 1)r_c}. \quad (8)$$

We can derive the effective temperature

$$T_{eff} = \frac{1}{2\pi x^2 A(x)} [Q^2 (x^6 + x^5 + x^4 - 2x^3 + x + 1) - x^2 (x^4 + x^3 - 2x^2 + x + 1)r_c^2], \quad (9)$$

where

$$A(x) = r_c^3 (x^2 + x + 1) [(x^3 - 1)f'(x) - 2x^2 f(x) - 2(x + 1)x]. \quad (10)$$

In the lukewarm case, there is

$$M^2 = Q^2 = \frac{x^2 r_c^2}{(x + 1)^2}. \quad (11)$$

With this, we can obtain the effective temperature of lukewarm RNdS black hole:

$$T_{eff} = -\frac{x(x^4 + 1)}{\pi(x + 1)^2(x^2 + x + 1)r_c A(x)}. \tag{12}$$

We also know that for the lukewarm RNdS black hole, the temperature is

$$T_+ = T_c = \frac{1 - x}{2\pi(x^2 + 1)r_c}. \tag{13}$$

Equating the two temperatures, we obtain a differential equation for $f(x)$:

$$f'(x) + \frac{2x^2}{1 - x^3} f(x) = \frac{2x^2(2x^5 + 3x^4 + 4x^3 - 2x - 3)}{(x^4 + x^3 - x - 1)^2}. \tag{14}$$

Fortunately, this equation has analytic solution, which is

$$f(x) = -\frac{2x}{5(x^4 + x^3 - x - 1)} \left[5x^5 + 3x^4 + 3x^3 - 5x^2 - 2x - 2 - 2(1 - x^3)^{2/3}(x^4 + x^3 - x - 1) \times {}_2F_1\left(\frac{1}{3}; \frac{2}{3}; \frac{4}{3}; x^3\right) \right]. \tag{15}$$

In Fig. 1, we depict the effective temperature T_{eff} and S as functions of x . It is shown that T_{eff} tends to zero as $x \rightarrow 1$, namely the charged Nariai limit. Although this result does not agree with that of Bousso and Hawking [13], it is consistent with the entropy. As is depicted in Fig. 1(b), the entropy will diverge as $x \rightarrow 1$. Besides, one can see the entropy is monotonically increasing with the increase of x , while T_{eff} first increases and then decreases. According to the general definition of heat capacity,

$$C = \frac{\partial M}{\partial T} = T \frac{\partial S}{\partial T},$$

only in the region of x with positive temperature and positive slope the RNdS black hole can be thermodynamically stable. This is unexpected. This means when the black hole horizon and the cosmological horizon are too far away (small x) or too close (large x), RNdS black hole cannot be thermodynamically stable.

In this letter, we have presented the entropy of RNdS black hole. It is not only the sum of the entropies of black hole horizon and the cosmological horizon, but also with an extra term from the correlation between the two horizons. This idea has twofold advantages. First, if without the extra term in the total entropy, the effective temperature is not the same as that of the black hole horizon and the cosmological horizon in the lukewarm case. This is not satisfactory. Second, the method of effective first law of thermodynamics lacks physical explanation or motivation. While taking advantage of the method, we obtain the corrected entropy of RNdS black hole, which may make the method more acceptable.

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